ANSWERS TO STUDY QUESTIONS

Chapter 13

13.1. Debt financing can create leverage by enabling the equity investor to purchase (and "own" or control) an asset whose value is many times greater than the amount of equity capital invested.

13.3. $LR = \frac{L}{E} + 1$

13.5. Equity's governance of the asset gives it the greater ability to influence the total value of the underlying asset (e.g., by management actions), while its residual claim causes any increment in this value to accrue to the value of the equity. If the equity owner manages the property well, they will reap the benefit; if they manage it poorly, they will suffer the loss (at least up to a point). Thus, by giving the equity owner the primary management control, it is likely that the value of the underlying asset (the property) will be maximized.

13.7. When debt is riskless, the risk and the risk premium in the equity return are both directly proportional to the leverage ratio (LR).

13.9. a. If the property is bought at fair market value and the loan is unsubsidized, leverage does not affect the equity investor's ex ante risk-adjusted return or expected risk premium per unit of risk.

b. Leverage does not affect the NPV of the equity investment provided the loan is unsubsidized and the investor's supply of equity capital is effectively unconstrained for the purpose of the relevant investment.

13.11. The expected total return includes the risk-free rate as well as the risk premium. Hence, the expected total return does not increase in proportion with the risk. Only the risk premium component of the total return increases with added risk brought by the leverage.

13.13. The underlying conceptual basis of the weighted average cost of capital (WACC) formula is the definition of the simple holding period return (HPR $=$ (CF Δ $+$ DV)/V) plus the combination of the value additivity principle: (*Assets* $=$ *Liabilities* $+$ *Owner's Equity*) and the income accounting identity (*Property cash flow* $=$ *Debt cash flow* $+$ *Equity cash flow*).

13.15. This statement is usually true on an ex ante basis, given that the expected total return to the mortgage is usually lower than the going-in property IRR. However, nothing requires that the mortgage must necessarily be less risky (as perceived by the capital markets) than the underlying property. Hence, it is possible, at least in principle, for mortgage interest rates to exceed property returns, resulting in negative leverage in the total return. Furthermore, if the equity investor overpays for the property, then the realistic expected return on the property is less than the market rate (unlevered), which could result in negative leverage.

13.17. If the property cap rate (net of capex) exceeds the mortgage constant, then the expected income component in the before-tax ex ante equity return will increase the greater is the loan-to-value (LTV) ratio. The expected income return will decrease with the LTV ratio if the cap rate net of capex is below the mortgage constant. If the net cap rate exactly equals the mortgage constant, then the equity income yield is invariant with respect to the LTV. The exact relationship can be seen in the WACC formula as discussed in sections 13.4 and 13.5.

13.19. Start with the ex ante relationship that links equity to property and debt returns, in either of the two forms, $r_E = r_D + (r_P - r_D)\,LR$, or, $r_E = r_p + (r_P - r_D)$ (L/E), expand the right-hand side and collect terms in r_P and then take the variance operator through.

$$r_E = r_D + (r_P - r_D)LR$$
$$r_E = r_P LR + r_D(1 - LR)$$
$$\text{var}[r_E] = LR^2\ \text{var}[r_P]$$
$$or\quad \sigma^2_{r_E} = \sigma^2_{r_P} LR^2$$
$$\Rightarrow\ \sigma_{r_E} = \sigma_{r_P}\quad LR = \sigma_{r_P}\left(1 + \frac{L}{E}\right)$$
$$or\ \sigma_{r_E} = \sigma_{r_P} + \sigma_{r_P}\left(\frac{L}{E}\right)$$

Property (or "business") risk

Added financial leverage risk

Notes …

$\text{Var}[r_D] = 0$, since the cost of debt is fixed and debt is assumed riskless in terms of default

σ^2 = var[.] {variance}

σ = √var[.] {standard deviation}

With a 75% LTV mortgage, the standard deviation of equity investor returns is (1 + 75/25) = 4 times higher than the standard deviation of underlying property returns.

13.21. Under the typical situation of nonnegative price appreciation in the property and nonnegative amortization in the loan, leverage will usually shift the expected return for the equity investor relatively away from the current income component and toward the growth or capital appreciation component.

13.23. Partners investing in the same underlying asset can attain their individual investment objectives through the use of mortgages (leverage). A partner requiring low-risk return would become the lender to the partnership and hence ascertain a fixed income on his/investment. The other partner with higher risk tolerance and low investment ability will get the residual income from the property (lender has superior lien) however by increasing the value of the property he/she can also get a higher return. Hence, leverage can be used to suit the investment requirements of different type of investors.

13.25.

	Pessimistic	Optimistic
No Leverage: $y_E = y_P =$ PBTCF/V	= 15/200 = 7.5%	= 25/200 = 12.5%
Leverage: $y_E =$ (PBTCF − DS)/E	= (15 − 10)/100 = 5%	(25 − 10)/100 = 15%

Levered returns are more volatile.

In the pessimistic (bad state of the world) case, the levered investor's return is less than the property return, whereas in the good state it is above the property return. It must be the case that ex post (after the fact) the property return (cap rate) is less than the mortgage constant (MC) in the bad state, but greater than the MC in the good state.

To confirm this, calculate the MC = DS/L = 10/100 = 10%.

Finally, we can show the leverage equation works. For example, in the pessimistic case

y_E = 7.5 + (7.5 − 10)(100/100) = 7.5 − 2.5 = 5%, similarly for the optimistic case.

13.27. a. LR = 100/25 = 4

(i) $NOI = 100 \times 0.075 = 7.5$, $DS = 0.75 \times 100 \times 0.0625 = 4.6875 \rightarrow y_E = (7.5 - 4.6875)/25 = 11.25\%$

b. (ii) $y_E = y_D + (LR)(y_P - y_D) = 6.25 + (7.5 - 6.25)(4) = 11.25\%$

13.29. a. With a property cap rate ($E[y_P]$) of 12% and expected total return of 10% ($E[r_P]$), the property's expected growth is $E[g_P]$ = 10% − 12% = 2% (remember, $g = r - y$). The mortgage growth rate must be −1%, as this is the difference between the interest rate (r_D = 10%) and MC (y_D = 11%). Thus, by the WACC:

$$\begin{aligned} E[g_E] &= E[g_D] + (LR)(E[g_P] - E[g_D]) \\ &= (-1\%) + [1/(1 - 80\%)][(-2\%) - (-1\%)] \\ &= (-1\%) + 5 \times (-1\%) = -6\% \end{aligned}$$

b. If the property cap rate were 9% and all else remained the same, the property would have expected growth of $+1\%$ (as $g = r - y = 10\% - 9\%$), so the equity growth would be

$$
\begin{aligned}
E[g_E] &= E[g_D] + (LR)(E[g_P] - E[g_D]) \\
&= (-1\%) + [1/(1 - 80\%)][1\% - (-1\%)] \\
&= (-1\%) + 5 \times (2\%) = 9\%
\end{aligned}
$$

13.31. With riskless debt, the equity risk is directly proportional to the leverage ratio: $LR = 1/(1 - LTV)$. Thus, with $LTV = 60\%$: $LR = 1/(1 - 0.6) = 1/0.4 = 2.5$; with $LTV = 80\%$, $LR = 1/(1 - 0.8) = 1/0.2 = 5$. So there is twice as much risk in the equity with the 80% LTV as compared to the 60% LTV (assuming riskless debt in both cases, i.e., ignoring the increase in the default risk for the debt holder). This would suggest twice the equity volatility or potential ex post return range (± 40% instead of − 20%), and this would require twice the risk premium in the equity return *ex ante*.

13.33. Apply the LTV-based WACC risk-premium formulation from equation (2):
$LTV = (RP_E - RP_P + /(RP_E - RP_D)$
Substitute data:

$$
\begin{aligned}
&RP_P = 3\% \\
&RP_D = r_D - r_f = 7\% - 5\% = 2\% \\
&\text{Target } RP_E = 7\% \\
&LTV = (7\% - 3\%)/(7\% - 2\%) \\
&LTV = 4\%/5\% = 80\%
\end{aligned}
$$