APPENDIX 14B

EXAMPLE APPLICATION OF APV TO AN INTRAMARGINAL INVESTOR

This Appendix picks up following section 14.3.6 in Chapter 14. To further refine your understanding of value additivity and the APV decision rule and to see their usefulness for intramarginal investment analysis, consider a pension fund. In the United States (as in many other countries), pension funds are effectively tax-exempt investors. (See the boxed feature *How Are Pension Fund Contributions Effectively Tax-Exempt?* on page 324 in Chapter 14.) This will normally make a pension fund an intramarginal investor in both the property and debt markets. So let us re-examine our levered investment in the $1 million apartment property of Chapter 14, only now assuming that the investor is a pension fund. For illustrative purposes we will hold all else the same, including the proposed use of the 5.5 percent, $750,000 loan.

How can the principle of value additivity help us to deepen our understanding of the pension fund's investment in the apartment property?[1] To see this, consider Exhibit 14B-1, which once again portrays the investor's "personalized" after-tax cash flows (in the tax-exempt pension fund's case, the same as its before-tax cash flows), together with separate evaluation of the property cash flows and the debt cash flows.

First, we determine the investment value to the pension fund of the property itself free and clear of debt. To do this, we discount the pension fund's "personalized" PATCFs (reflecting the fund's zero tax rate) at the market's unlevered (property level) after-tax OCC that we derived in section 14.3.6 in Chapter 14, from analysis of the marginal taxed investor in the property market. There (as summarized in Exhibit 14-6) we determined that the after-tax going-in IRR for unlevered investment in the apartment property was 4.76 percent. Assuming that the tax rates in Exhibit 14-3 (35 percent on ordinary income, 15 percent on capital gain, and 25 percent on depreciation recapture) typify the marginal investor in the market for such apartment property (and assuming that $1 million is the market value of such property), 4.76 percent represents the market's after-tax OCC for the subject property.[2] This results in an IV of the property to the tax-exempt pension fund of $1,104,714, considerably above the $1 million MV of the property, reflecting the fund's tax advantage compared to marginal investors in the apartment market.

Similarly, we determine that the investment value to the pension fund of the future after-tax cash outflows on the loan is negative $832,202. This is derived from discounting the loan's after-tax payments (for the P.F. the same as the before-tax debt service) at the market's after-tax OCC for the debt, based on a comparison of municipal bond yields versus corporate bond yields as described in section 14.3.6. Since the pension fund faces no income taxes, it obtains no tax shield from the interest expense on the loan, causing the loan interest payments to have a greater present value to the pension fund than to the marginal investor in the debt market, who is taxed (and therefore obtains value from the interest tax shield).

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[1] In mainstream corporate finance, the value additivity and APV principles are often derived using an arbitrage analysis which is in theory applicable only to market value. However, the fundamental principle of value additivity applied to cash flow components can also be applied to analyze investment value, taking care as always to apply appropriate market-based OCC discount rates to each personalized cash flow component.

[2] Recall our discussion in section 14.3.5 (and earlier in section 12.1 of Chapter 12) that the appropriate discount rate for IV valuation should usually come from the capital market. See in particular footnote 30 in section 14.3.5.

Apprec.Rate =	1.00%
Yield =	6.00%
Income Tax Rate =	0.00%
Debt Mkt Margl Tax Rate =	25.00%

Bldg.Val/Prop.Val =	80.00%
Depreciable Life =	27.5 years
CGTax Rate =	0.00%
DepRecapture Rate =	0.00%

Loan =	$750,000
Int =	5.50%
Amort/yr	$2,000

PV @ 6.44% = $270,548

Year	(1) Prop.Val	(2) NOI	(3) CI	(4) PBTCF	(5) tax w/out shields	(6) DTS	(7) (4)−(5)+(6) PATCF	(8) LoanBal	(9) Loan DS	(10) ITS	(11) (4)−(9) EBTCF	(12) (7)−(9)+(10) EATCF	(13) (9)−(10) LoanATCFs
0	$1,000,000			($1,000,000)			($1,104,714)	$750,000	($750,000)		($250,000)	($270,548)	($832,202)
1	$1,010,000	$60,000	$0	$60,000	$0	$0	$60,000	$748,000	$43,250	$0	$16,750	$16,750	$43,250
2	$1,020,100	$60,600	$0	$60,600	$0	$0	$60,600	$746,000	$43,140	$0	$17,460	$17,460	$43,140
3	$1,030,301	$61,206	$50,000	$11,206	$0	$0	$11,206	$744,000	$43,030	$0	($31,824)	($31,824)	$43,030
4	$1,040,604	$61,818	$0	$61,818	$0	$0	$61,818	$742,000	$42,920	$0	$18,898	$18,898	$42,920
5	$1,051,010	$62,436	$0	$62,436	$0	$0	$62,436	$740,000	$42,810	$0	$19,626	$19,626	$42 810
6	$1,061,520	$63,061	$0	$63,061	$0	$0	$63,061	$738,000	$42,700	$0	$20,361	$20,361	$42,700
7	$1,072,135	$63,691	$0	$63,691	$0	$0	$63,691	$736,000	$42,590	$0	$21,101	$21,101	$42,590
8	$1,082,857	$64,328	$50,000	$14,328	$0	$0	$14,328	$734,000	$42,480	$0	($28,152)	($28,152)	$42,480
9	$1,093,685	$64,971	$0	$64,971	$0	$0	$64,971	$732,000	$42,370	$0	$22,601	$22,601	$42,370
10	$1,104,622	$65,621	$0	$1,170,243	$0	$0	$1,170,243	$730,000	$772,260	$0	$397,983	$397,983	$772,260
IRR of above CF Stream =				6.04%			4.76%		5.50%		7.40%	6.44%	4.13%

PV @ 4.76% = $1,104,714

PV @ 4.13% = $832,202

Value Additivity:
V − D = E
IV(equity) = $1,104,714 − $832,202 = $272,512 ≈ $270,548*

*In principle this equation should be exactly equal, otherwise there is some sort of "arbitrage" opportunity between the markets. But in reality valuations are not that precise.

EXHIBIT 14B-1
Investment Value by Component Additivity for Intramarginal Tax-exempt Investor: Equity Value Equals Property Value Minus Debt Value

Invoking the value additivity principle, the pension fund's levered equity position in the investment consists of the property value minus the loan value: $\$1,104,714 - \$832,202 = \$272,512$. This gives us some additional insight.

Applying the APV procedure, and assuming we could buy the property for its estimated market value of $1,000,000, we see that the deal for the pension fund consists of a positive-NPV component combined with a negative NPV component:

$$\begin{aligned} \boldsymbol{APV} &= \boldsymbol{NPV}(\boldsymbol{Property}) + \boldsymbol{NPV}(\boldsymbol{Financing}) \\ APV &= (\$1{,}104{,}714 - \$1{,}000{,}000) + (\$750{,}000 - \$832{,}202) \\ APV &= 104{,}714 + (-\$82{,}202) = \$22{,}512 \end{aligned}$$

Purchasing the property without any debt brings the pension fund a positive NPV of $104,714, while borrowing $750,000 causes the pension fund a negative NPV of $82,202. We see through the APV analysis of this intramarginal investor that it would make more sense for the pension fund to buy the property without using debt. This should not be surprising after our analysis in section 14.3.5 of the value of debt financing for investors that face different tax rates. The pension fund clearly faces a tax rate lower than that of the marginal investor in the debt market, and so faces a positive NPV from debt investment, hence a negative NPV from "negative investment" in debt, that is, borrowing.[3]

Finally, let's step back and see what this analysis tells the pension fund. First, it suggests that if they make the apartment investment without using debt, they could pay up to just over $1.1 million for the property and the investment would still be positive NPV from their investment value perspective. If they use 75 percent debt of the terms in our example, they can only pay up to about $1.02 million and still achieve positive investment value NPV. Of course, the pension fund should not have to pay more than the market value of $1 million for the apartment property, so what this really tells them is that they can make a positive NPV of about $100,000 if they don't use debt financing, or only about $20,000 if they use the 75 percent LTV loan. The leverage would allow them to buy four equivalent properties instead of just one, so for the same $1,000,000 of investable capital, the leveraged approach would produce a positive NPV of over $80,000, which is probably not significantly less than the nonleveraged approach. Of course, the leveraged approach will involve more risk, including more volatility in the return, as they move along the risk/return trade-off frontier we discussed in Chapter 13. The pension fund will need to weigh these countervailing considerations from a strategic and tactical investment policy perspective to decide what they want to do.

[3]In the real world, this conclusion is tempered by the fact that the pension fund is capital constrained. The pension fund cannot issue new equity like a taxed corporation could. Its available funds are exogenously fixed. The availability of tax-sheltered investment capital is effectively constrained by the government's tax laws governing retirement savings and investments.